

Appendix A. Proof of Theorem 1

In this section we present the proof of theorem 1.

Theorem 1. *Let E_1 and E_2 be the set of solutions to Sys1 and Sys2 below.*

$$\max_{C1, C3, C5-C8, C10} (4) \quad (\text{Sys1})$$

$$\max_{C1-C11} (4) \quad (\text{Sys2})$$

We have

$$E1 = E2. \quad (\text{Th})$$

Proof. We say that $v \in E_1$ is a solution that satisfies Sys1 and we write $v \vdash \text{Sys1}$. We first prove that all solutions of Sys2 are solutions to Sys1 ($E_1 \subseteq E_2$). Then we prove the other direction ($E_2 \subseteq E_1$).

Appendix A.1. Left: $E_1 \subseteq E_2$

We claim

$$\forall v \in E_1. v \vdash C2, C4, C9. \quad (\text{A.1})$$

We rewrite A.1 as

$$\neg \exists v \in E_1. (v \not\vdash C2) \vee (v \not\vdash C4) \vee (v \not\vdash C9), \quad (\text{Th}')$$

and we show Th' case by case.

Case: $\neg \exists v \in E_1. v \not\vdash C2$. Recall that C2 ensures that projects that are not selected in a quarter, have no resources allocated to them during that quarter. Assume $v \not\vdash C2$, thus v is configured such that

$$\exists p, t. \mathbf{S}^v[p][t] = 0 \wedge \exists k. \mathbf{A}^v[t][p][k] > 0. \quad (\text{A.2})$$

We construct the solution u exactly similar to v except for the following.

$$\forall p, t. \mathbf{S}^u[p][t] = 0 \wedge \forall k. \mathbf{A}^u[t][p][k] = 0. \quad (\text{A.3})$$

We have $u \vdash C1, C3, C5 - C8, C10$. We also have $(4)^u > (4)^v$ since with u we reduce effort without increasing cost or reducing revenue. Thus $v \notin E_1$.

Case: $\neg \exists v \in E_1. v \not\vdash C4$. Recall that C4 guarantees that we do not work on projects that are not selected. Assume $v \not\vdash C4$, thus v is configured such that

$$\exists p. \boldsymbol{\gamma}^v[p] = 0 \wedge \sum_{t=1}^T \mathbf{S}^v[p][t] > 0 \quad (\text{A.4})$$

This is equivalent to

$$\exists p, (\gamma^v[p] = 0) \wedge (\exists t, \mathbf{S}^v[p][t] = 1) \quad (\text{A.5})$$

We substitute for \mathbf{S}^v with its implications using C1 to obtain

$$\begin{aligned} \exists p, (\gamma^v[p] = 0) \wedge (\exists t, \forall j, \mathbf{B}^v[t][p][j] \geq \mathbf{W}^v[p][j]) \\ \wedge (\exists j, \mathbf{W}^v[p][j] > 0). \end{aligned} \quad (\text{A.6})$$

We construct u that is exactly similar to v except for the following.

$$\forall p. \gamma^v[p] = 0 \Rightarrow \forall t, j. \mathbf{B}^u[t][p][j] = 0. \quad (\text{A.7})$$

We have $u \vdash C1, C3, C5 - C8, C10$. We also have $(4)^u > (4)^v$ since with u we reduce effort without increasing cost or reducing revenue. Thus $v \notin E_1$.

Case: $\neg \exists v \in E_1. v \not\vdash C10$. Recall that C9 guarantees that if we are busy in one quarter, then there is at least one project selected in that quarter. Assume $v \not\vdash C9$, thus v is configured such that

$$\exists t, (\mathbf{b}^v[t] = 1) \wedge (\forall p, \mathbf{S}^v[p][t] = 0). \quad (\text{A.8})$$

We construct u that is exactly similar to v except for the following.

$$\forall p, t. \mathbf{S}^u[p][t] = 0 \Rightarrow \mathbf{b}^u[t] = 0. \quad (\text{A.9})$$

We have $u \vdash C1, C3, C5 - C8, C10$. We also have $(4)^u > (4)^v$ since with u we reduce cost without increasing effort or reducing revenue. Thus $v \notin E_1$. We have proved all cases of Th' and thus $E_1 \subseteq E_2$.

Appendix A.2. Right: $E_2 \subseteq E_1$

All solutions of Sys2 satisfy all constraints of Sys1 as they satisfy more constraints. We need to prove that they actually maximize (4). The three cases of $E_1 \subseteq E_2$ show that satisfying the constraints C2, C4, and C9 after satisfying all other constraints, reduces cost or effort monotonically without decreasing revenue. This concludes our proof. \square